

Quarkonium transport in thermal AdS/CFT

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ABSTRACT: We consider a heavy meson moving slowly through high temperature non-Abelian plasmas. Using a simple dipole effective Lagrangian, we calculate the in-medium mass shift and the drag coefficient of the meson in $\mathcal{N} = 4$ Super Yang Mills theory at weak and strong coupling. As anticipated, in the large N limit the mass shift is finite while the drag is suppressed by $1/N^2$. After comparing results to perturbative QCD estimates (which are also calculated), we reach the conclusion that relative to weak coupling expectations the effect of strong coupling is to reduce the momentum diffusion rate and to *increase* the relaxation time by up to a factor of four.

KEYWORDS: Gauge-gravity correspondence, AdS-CFT Correspondence, Thermal Field Theory, Supersymmetric Effective Theories.

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1. Introduction

The energy loss of heavy quarks and quarkonia in media has been a subject of intense experimental interest [1–7]. The suppression of charm and bottom quarks observed at RHIC motivated several groups to utilize the gauge-gravity duality [8–11] to compute the drag of fundamental heavy quarks in $\mathcal{N} = 4$ Super Yang-Mills at strong coupling [12–14]. The goal of this paper is to extend these strong coupling calculations of drag and diffusion from heavy quarks to heavy mesons.

The motivation for this effort is twofold. First, future experiments at RHIC promise to measure the elliptic flow of J/ψ mesons, and it is important to support this experimental program with theoretical work. To this end, various groups have studied the thermal properties of heavy mesons within the context of the AdS/CFT correspondence [15–18]. However, in spite of this progress, the transport properties of these mesonic excitations are not well understood. Although the kinetics derived in this paper are not directly applicable to the heavy ion experiments, we believe that the results do hold some important information for phenomenology.

The second motivation for this work is theoretical. After the quark drag was computed using the correspondence, it was realized that the drag of quarkonia is zero in a large N_c limit [15, 16, 18]. In the high temperature phase (when the temperature is comparable to the mass of lowest meson state) the vector spectral function has been computed and it shows a rich dynamical picture of meson melting [19–23]. However, in the low temperature phase describing heavy mesons, the spectral function is usually described by a sequence of states with zero width. Since within a thermal environment the drag and diffusion of these mesonic states is certainly not zero, it remained as a theoretical challenge to compute the kinetics of these states using the AdS/CFT setup.

Rising to this challenge, the width of AdS/CFT mesons due to scattering with surrounding heavy quarks (or anti-quarks) was recently determined by extending the analysis of meson melting to finite baryon density [19, 16] and by studying string worldsheet instantons at zero baryon density [24]. In general the meson width determined in this way is suppressed by the density of heavy quarks. (At zero baryon density the width is suppressed by the thermal population of heavy quarks.) In contrast, we are concerned with the thermal width which is finite at zero density and infinite λ and captures the rescattering between the meson and the surrounding $\mathcal{N} = 4$ medium.

This work will focus on heavy mesons where the binding energy is much greater than the temperature. While in perturbation theory the constraint on the binding energy reads $m_q v^2 \gg T$, in the strongly coupled $\mathcal{N} = 4$ theory the constraint is $\frac{2\pi m_q}{\sqrt{\lambda}} \gg T$. In this tight binding regime, mesons survive well above T_c and the meson width is sufficiently narrow to speak sensibly about drag and momentum diffusion.

For real charmonium (bottomonium) the binding energy can be estimated from mass splitting between the $2s$ and $1s$ ($3s$ and $1s$) states, $\Delta M_{2s-1s}^{J/\psi} \simeq 589$ MeV and $\Delta M_{3s-1s}^{\Upsilon} \simeq 895$ MeV respectively [25]. Therefore it is not really clear that real quarkonia above $T_c \simeq 170 - 190$ MeV [26, 27] can be modeled as a simple dipole which lives long enough to be considered a quasi-particle. Indeed weak coupling hot QCD calculations of the spectral function show that over the temperature range $g^2 M - gM$, the concept of a meson quasi-particle slowly transforms from being well defined to being increasingly vague [28–32]. There is lattice evidence based on the maximal entropy method (which is not without uncertainty) that J/ψ and Υ survive to $1.6 T_c$ and $\sim 3 T_c$ respectively [33–38]. However, model potential calculations which fit all the Euclidean lattice correlators indicate that the J/ψ and Υ survive only up to at most $1.2 T_c$ and $2.0 T_c$ respectively [39, 40]. Clearly, the word “survive” in this context is qualitative and means that there is a discernible peak in the spectral function. Given these facts, the assessment of the authors is that the dipole approximation might be reasonable for $\Upsilon(1S)$ but poor for charmonium states and other bottomonium states.

Within the context of gauge gravity duality, mesons are studied by exploiting convenient generalizations of the AdS/CFT correspondence in which fundamental flavor degrees of freedom are added by inserting additional probe branes into the geometry. In this paper we will use the approach due to Karch and Katz [41] in which the additional quark flavors are obtained by adding D7 brane probes to the original D3 brane setup. On the gravity side, the probe branes wrap a subspace which asymptotically near the boundary is

$AdS_5 \times S^3$. The meson spectrum for the dual $\mathcal{N} = 2$ supersymmetric gauge theory was first calculated in [42] by considering fluctuations of the D7 branes embedded. Restricting this result to fluctuations with vanishing angular momentum on the S^3 , the meson spectrum is given by

$$M = \frac{2\pi m_q}{\sqrt{\lambda}} 2\sqrt{(n+1)(n+2)}, \quad (1.1)$$

with m_q the quark mass determined by the separation between the D3 and D7 branes, λ the 't Hooft coupling and n the radial excitation number. Subsequently, a gravity dual of chiral symmetry breaking has been obtained in [43] by embedding a D7 brane probe into a deformed non-supersymmetric gravity background with non-trivial dilaton [44]. In this case there is a Goldstone boson in the meson spectrum. The thermodynamics of mesons has been studied within gauge/gravity duality by embedding a D7 brane probe into the AdS-Schwarzschild black hole background. Within the deconfined phase, there is a new fundamental first order phase transition which corresponds to meson melting [43, 45–47, 21]. For a review on mesons in the AdS/CFT correspondence see [48].

In this paper we consider a heavy meson moving slowly through the medium. We perform both a perturbative QCD and a strong coupling $\mathcal{N} = 4$ SYM computation. For both approaches we first calculate the in medium meson mass shift, which determines the polarizabilities of the meson. As expected from the dipole effective theory, the mass shift scales as T^4/Λ_B^3 , with T the temperature and Λ_B the inverse size of the meson. In the perturbative calculation, Λ_B is the inverse Bohr radius, while in the AdS/CFT computation the meson mass plays this role. In the $\mathcal{N} = 4$ field theory the dipole effective Lagrangian couples the heavy meson to the stress tensor and the square of the field strength \mathcal{O}_{F^2} . In AdS/CFT we obtain these couplings from the linear response of the meson mass to switching on a black hole background or a non-trivial dilaton flow, respectively. For the dilaton flow we consider the $D3+D(-1)$ gravity background of Liu and Tseytlin [49]. This background and the AdS-Schwarzschild background allow for an analytic calculation of the meson polarizabilities.

Using these polarizabilities we subsequently calculate the momentum broadening κ and the drag coefficient η_D . This requires the calculation of two-point functions involving gradients of the stress tensor and the field strength squared. Within gauge/gravity duality, these are obtained by considering graviton and dilaton propagation through the AdS-Schwarzschild black hole background.

An outline of the paper is as follows. First, in section 2 we review the computation of drag and diffusion of heavy $Q\bar{Q}$ bound states within the setup of perturbative QCD. This will outline a two step procedure to determine the drag coefficient at strong coupling. The first step is to determine the in medium mass shift (which is finite at large N_c) which determines the polarizabilities of the meson. This is done in section 3. The second step is to compute the force-force correlator of the meson using the previously computed polarizabilities. This determines the drag and diffusion coefficient as reviewed in section 4. Finally we compare our results to perturbation theory and reach some conclusions for the RHIC experiments in section 5.

2. Diffusion of heavy mesons in perturbative large N field theories

2.1 Diffusion of heavy mesons in perturbative large N QCD

The interactions of a heavy meson with the QCD medium is well described by a dipole approximation. This physical approximation has been formalized in the language of heavy meson effective Lagrangians which we will adopt [50]. This perturbative scheme relies on the large mass of the meson relative to the external momenta of the gauge fields but does not rely on the smallness of the coupling constant. It was used previously to make a good estimate for the binding of J/ψ to nuclei [50].

The heavy meson field ϕ_v describes a (scalar) meson which has a fixed velocity $v^\mu = (\gamma, \gamma\mathbf{v})$. Then the effective Lagrangian for this meson field interacting with the gauge fields is

$$\mathcal{L}_{\text{eff}} = -\phi_v^\dagger i v \cdot \partial \phi_v + \frac{c_E}{N^2} \phi_v^\dagger \mathcal{O}_E \phi_v + \frac{c_B}{N^2} \phi_v^\dagger \mathcal{O}_B \phi_v, \quad (2.1)$$

with

$$\mathcal{O}_E = -\frac{1}{2} G^{\mu\alpha A} G_\alpha{}^{\nu A} v_\mu v_\nu, \quad \mathcal{O}_B = \frac{1}{4} G^{\alpha\beta A} G_{\alpha\beta}{}^A - \frac{1}{2} G^{\mu\alpha A} G_\alpha{}^{\nu A} v_\mu v_\nu. \quad (2.2)$$

$G^{\mu\nu}$ is the non-Abelian field strength of QCD, and c_E and c_B are matching coefficients (polarizabilities) to be determined from the QCD dynamics of the heavy $Q\bar{Q}$ pair. In inserting a factor of $1/N^2$ into the effective Lagrangian we have anticipated that the couplings of the heavy meson to the field strengths are suppressed by N^2 in the large N limit.

In the rest frame of a heavy quark bound state, $v = (1, 0, 0, 0)$, the operators \mathcal{O}_E and \mathcal{O}_B are

$$\mathcal{O}_E = \frac{1}{2} \mathbf{E}^A \cdot \mathbf{E}^A, \quad \mathcal{O}_B = \frac{1}{2} \mathbf{B}^A \cdot \mathbf{B}^A, \quad (2.3)$$

where \mathbf{E}^A and \mathbf{B}^A are the color electric and magnetic fields. If the constituents of the dipole are non-relativistic it is expected that the magnetic polarizability c_B is $O(v^2)$ relative to the electric polarizability. For heavy quarks (where c_B is neglected) and large N these matching coefficients were computed by Peskin [51, 52]

$$c_E = \frac{28\pi}{3\Lambda_B^3}, \quad c_B = 0. \quad (2.4)$$

Here $\Lambda_B \equiv 1/a_0 = (m_q/2)C_F\alpha_s$ is the inverse Bohr radius for a $Q\bar{Q}$ bound state. It is finite at large N since with $C_F \simeq N/2$ and finite λ we have $\Lambda_B = m_q\lambda/16\pi$. We will assume that $c_B = 0$ is zero in this section and subsequently generalize our results to $\mathcal{N} = 4$ theory.

The effective Lagrangian can be used to compute both the thermodynamics and kinetics of the heavy meson state. First, to evaluate the in medium mass shift one simply uses first order perturbation theory $\delta M = \langle H_1 \rangle = -\langle \mathcal{L}_1 \rangle$, yielding

$$\begin{aligned} \delta M &= -\frac{c_E}{N^2} \langle \mathcal{O}_E \rangle_T \\ &= -T \left(\frac{\pi T}{\Lambda_B} \right)^3 \frac{14}{45}. \end{aligned} \quad (2.5)$$

In the second line we have simply calculated the expectation value $\langle \mathcal{O}_E \rangle_T = \frac{\pi^2}{30} N^2 T^4$ in a free gluon gas and used eq. (2.4).

The importance of this result is that it is finite at large N and that it is in general suppressed by $(T/\Lambda_B)^3$, i.e. suppressed by powers of the hadron scale to the temperature. If higher dimension operators were added to the effective Lagrangian their contributions would be suppressed by additional powers of T/Λ_B . At strong coupling, we will use the AdS/CFT correspondence to determine the $\mathcal{N} = 4$ polarizabilities from the mass shift.

We turn next to the kinetics of a heavy dipole in the medium. For time scales which are long compared to medium correlations, we expect that the kinetics of the heavy meson is described by Langevin equations

$$\frac{dp_i}{dt} = \xi_i(t) - \eta_D p_i, \quad \langle \xi_i(t) \xi_j(t') \rangle = \kappa \delta_{ij} \delta(t - t'). \quad (2.6)$$

Here ξ_i is a random force with second moment κ and η_D is the drag coefficient. The drag and fluctuation coefficients are related through the Einstein relation

$$\eta_D = \frac{\kappa}{2MT}. \quad (2.7)$$

The Langevin equation is valid for times which are long compared to the inverse temperature but short compared to the lifetime of the quasi-particle state.

We can use the effective dipole Lagrangian to calibrate the noise of the stochastic evolution, κ . The microscopic equations of motion for a heavy particle in the medium are

$$\frac{dp^i}{dt} = \mathcal{F}^i(t), \quad (2.8)$$

where \mathcal{F} is a phenomenological force to be specified below. We then compare the response of the Langevin process to the microscopic theory. Over a time which is long compared to medium correlations but short compared to the time scale of equilibration we can neglect the drag and equate the stochastic process to the microscopic theory

$$\int dt \int dt' \langle \xi_i(t) \xi_j(t') \rangle = (\text{time}) \times \kappa \delta_{ij} = \int dt \int dt' \langle \mathcal{F}_i(t) \mathcal{F}_j(t') \rangle. \quad (2.9)$$

In a rotationally invariant medium we have

$$\kappa = \frac{1}{3} \int dt \langle \mathcal{F}^i(t) \mathcal{F}^i(0) \rangle. \quad (2.10)$$

In the present context we identify the force with the negative of the gradient of the interaction Hamiltonian $H_I = -\mathcal{L}_I$,

$$\mathcal{F}(t) = \int d^3\mathbf{x} \phi_v^\dagger(\mathbf{x}, t) [c_E \nabla \mathcal{O}_E(\mathbf{x}, t)] \phi_v(\mathbf{x}, t), \quad (2.11)$$

which is the usual form of a dipole force averaged over the wave function of the meson. The fluctuation dissipation theorem relates the correlation function in eq. (2.10) (with the specified time order of operators) to the imaginary part of the retarded force-force correlator

$$\kappa = \frac{1}{3} \lim_{\omega \rightarrow 0} \frac{-2T}{\omega} \text{Im} G_R(\omega), \quad (2.12)$$

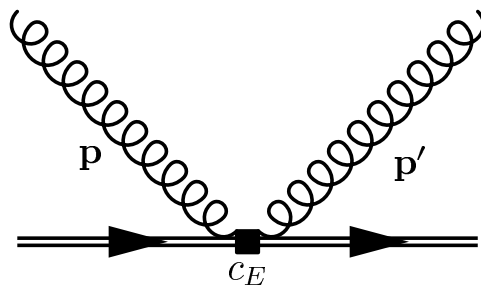


Figure 1: Dipole scattering graph which causes drag and diffusion of heavy mesons in QCD and $\mathcal{N} = 4$ SYM. The QCD correlator which encodes this physics is given by eq. (2.14).

where the retarded correlator is

$$G_R = -i \int dt e^{+i\omega t} \theta(t) \langle [\mathcal{F}^i(t), \mathcal{F}^i(0)] \rangle . \quad (2.13)$$

Integrating out the heavy meson field as discussed in detail in ref. [13], which treated the heavy quark case, we obtain a formula for the momentum diffusion coefficient

$$\kappa = \frac{1}{3} \frac{c_E^2}{N^4} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \mathbf{q}^2 \left[-\frac{2T}{\omega} \text{Im} G_R^{\mathcal{O}_E \mathcal{O}_E}(\omega, \mathbf{q}) \right] , \quad (2.14)$$

with the retarded $\mathcal{O}_E \mathcal{O}_E$ correlator given by

$$G_R^{\mathcal{O}_E \mathcal{O}_E}(\omega, \mathbf{q}) = -i \int d^4x e^{+i\omega t - i\mathbf{q}\cdot\mathbf{x}} \theta(t) \langle [\mathcal{O}_E(\mathbf{x}, t), \mathcal{O}_E(\mathbf{0}, 0)] \rangle . \quad (2.15)$$

We can understand this result with simple kinetic theory. Examining the Langevin dynamics we see that 3κ is the mean squared momentum transfer to the meson per unit time. The factor of three arises from the number of spatial dimensions. In perturbation theory this rate is easily computed by weighting the transition rate for any gluon in the bath to scatter with the heavy quark by the square of the momentum transfer,

$$3\kappa = \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_{\mathbf{p}}} \frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{\mathbf{p}'}} |\mathcal{M}|^2 n_{\mathbf{p}} (1 + n_{\mathbf{p}'}) \mathbf{q}^2 (2\pi)^3 \delta^3(\mathbf{q} - \mathbf{p} + \mathbf{p}') . \quad (2.16)$$

Here \mathbf{p} is the incoming gluon, \mathbf{p}' is the outgoing gluon and \mathbf{q} is the momentum transfer $\mathbf{q} = \mathbf{p} - \mathbf{p}'$ as indicated in figure 1.

$|\mathcal{M}|^2$ is the gluon meson scattering amplitude computed with the effective Lagrangian in eq. (2.1) and summed over colors and helicities of the incoming and outgoing gluon

$$|\mathcal{M}|^2 = \frac{c_E^2}{N^2} \omega^4 (1 + \cos^2(\theta_{\mathbf{p}\mathbf{p}'}) . \quad (2.17)$$

Alternatively (as detailed in appendix A) we can simply evaluate the imaginary part of the retarded amplitude written in eq. (2.14) to obtain the same result.

For QCD the integrals written in eq. (2.16) are straightforward and yield the following result for the rate of momentum broadening

$$\begin{aligned} \kappa &= \frac{1}{N^2} c_E^2 \frac{64\pi^5}{135} T^9 \\ &= \frac{T^3}{N^2} \left(\frac{\pi T}{\Lambda_B} \right)^6 \frac{50176\pi}{1215} . \end{aligned} \quad (2.18)$$

The high power of temperature T^9 arises since the dipole cross section rises as ω^4 . The matching coefficient c_E is directly related to the mass shift of the dipole and encodes the coupling of the long distance gluonic fields to the dipole. By taking the ratio between the momentum broadening and the mass shift squared we find a physical quantity which is independent of this coupling

$$\frac{\kappa}{(\delta M)^2} = \frac{\pi T}{N^2} \frac{1280}{3} . \quad (2.19)$$

The large numerical factor 1280/3 originates from the cross section which grows as ω^4 . A similarly large factor appears in $\mathcal{N} = 4$ SYM as discussed below in greater detail.

2.2 Linear perturbations of $\mathcal{N} = 4$ super Yang-Mills theory

Our aim is to calculate the heavy meson diffusion coefficient κ from gauge/gravity duality. This requires the calculation of the two-point correlators as well as of the associated polarizabilities in $\mathcal{N} = 4$ Super Yang-Mills theory.

The same formalism used in the preceding section can be used for $\mathcal{N} = 4$ $SU(N)$ Super Yang-Mills theory. In general all operators in $\mathcal{N} = 4$ which are scalars under Lorentz transformations and $SU(4)$ R-charge rotations will couple to the meson at some order. The contribution of higher dimensional operators is suppressed by powers of the temperature to the inverse size of the meson. The lowest dimension operator which could couple to the heavy meson field is $\mathcal{O}_{X^2} = \text{tr} X^i X^i$, where X^i denotes the scalar fields of $\mathcal{N} = 4$ theory. However the anomalous dimension of this operator is not protected, and the prediction of the supergravity description of $\mathcal{N} = 4$ SYM is that these operators decouple in a strong coupling limit [11]. The lowest dimension gauge invariant local operators which are singlets under $SU(4)$ and which have protected anomalous dimension are the stress tensor $T_{\mu\nu}$ which couples to the graviton, minus the Lagrangian $\mathcal{O}_{F^2} = -\mathcal{L}_{\mathcal{N}=4}$, which couples to the dilaton¹ and the operator $\mathcal{O}_{F\tilde{F}} = \text{tr} F^{\mu\nu} \tilde{F}_{\mu\nu} + \dots$, which couples to the axion. An interaction involving $\mathcal{O}_{F\tilde{F}}$ breaks CP which is a symmetry of the Lagrangian $\mathcal{N} = 2$ hypermultiplet of the $\mathcal{N} = 4$ SYM gauge theory. Thus interactions involving $\mathcal{O}_{F\tilde{F}}$ can be neglected.

Summarizing the preceding discussion, we find that the effective Lagrangian describing the interactions of a heavy meson coupling to the operators in the field theory is

$$\mathcal{L}_{\text{eff}} = -\phi_v(\mathbf{x}, t) i v \cdot \partial \phi_v(\mathbf{x}, t) + \frac{c_T}{N^2} \phi_v^\dagger(\mathbf{x}, t) T^{\mu\nu} v_\mu v_\nu \phi_v(\mathbf{x}, t) + \frac{c_F}{N^2} \phi_v^\dagger(\mathbf{x}, t) \mathcal{O}_{F^2} \phi_v(\mathbf{x}, t) , \quad (2.20)$$

which is a linear perturbation of $\mathcal{N} = 4$ Super Yang-Mills theory by two composite operators.

The polarization coefficients c_T , c_F will be determined below from meson mass shifts in gauge/gravity duality. This requires breaking some of the supersymmetry. For the contribution of the energy-momentum tensor, this is achieved by switching on the temperature. Then, the mass shift of the meson is given by expectation value of the stress tensor

$$\delta M = -\frac{c_T}{N^2} \langle T^{00} \rangle , \quad (2.21)$$

¹Since we can add a total derivative to the Lagrangian, the operator $-\mathcal{L}$ is ambiguous. The precise form of the operator coupling to the dilaton is given in ref. [53]. We neglect this ambiguity here.

In gauge/gravity duality this is achieved by considering the AdS-Schwarzschild black hole background where $\langle \mathcal{O}_{F^2} \rangle = 0$. In contrast, for the meson response to $\langle \mathcal{O}_{F^2} \rangle$ we consider a background self-dual gauge configuration where $\langle \mathcal{O}_{F^2} \rangle \neq 0$ while $\langle T^{\mu\nu} \rangle = 0$. As can be seen from the supersymmetry transformations of the $\mathcal{N} = 4$ fermions, such a background breaks the supersymmetry to $\mathcal{N} = 2$. The mass shift of a heavy meson is then

$$\delta M = -\frac{c_F}{N^2} \langle \mathcal{O}_{F^2} \rangle, \quad (2.22)$$

where $\langle \mathcal{O}_{F^2} \rangle = \langle -\mathcal{L} \rangle$ is the expectation value of the $\mathcal{N} = 4$ Lagrangian in the $\mathcal{N} = 2$ symmetric background configuration. A suitable dilaton solution corresponding to this self-dual configuration been given by Liu and Tseytlin [49] and will be used below to determine c_F .

As explained above for the QCD case, to determine the kinetics of the heavy meson in the $\mathcal{N} = 4$ background we identify the force on the heavy meson as minus the gradient of the interaction Hamiltonian $H_I = -\mathcal{L}_I$ averaged over the meson wave function. With eq. (2.20) we have

$$\mathcal{F}(t) = \int d^3 \mathbf{x} \phi_v^\dagger(\mathbf{x}, t) \nabla \left[\frac{c_T}{N^2} T^{\mu\nu} u_\mu u_\nu + \frac{c_{F^2}}{N^2} \mathcal{O}_{F^2} \right] \phi_v(\mathbf{x}, t). \quad (2.23)$$

Then integrating out the heavy meson fields as above, the force-force correlator in eq. (2.12) becomes

$$\kappa = \lim_{\omega \rightarrow 0} \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{\mathbf{q}^2}{3} \left[\left(\frac{c_T}{N^2} \right)^2 \frac{-2T}{\omega} \text{Im} G_R^{TT}(\omega, \mathbf{q}) + \left(\frac{c_F}{N^2} \right)^2 \frac{-2T}{\omega} \text{Im} G_R^{F^2 F^2}(\omega, \mathbf{q}) \right], \quad (2.24)$$

where the retarded correlators are

$$G_R^{TT} = -i \int d^4 x e^{+i\omega t - i\mathbf{q}\cdot\mathbf{x}} \theta(t) \langle [T^{00}(\mathbf{x}, t), T^{00}(\mathbf{0}, 0)] \rangle, \quad (2.25)$$

$$G_R^{F^2 F^2} = -i \int d^4 x e^{+i\omega t - i\mathbf{q}\cdot\mathbf{x}} \theta(t) \langle [\mathcal{O}_{F^2}(\mathbf{x}, t), \mathcal{O}_{F^2}(\mathbf{0}, 0)] \rangle. \quad (2.26)$$

In writing eq. (2.24) we have implicitly assumed that there is no cross term between \mathcal{O}_{F^2} and $T^{\mu\nu} v_\mu v_\nu$. In the gauge/gravity duality this is reflected in the fact that at tree level in supergravity, $\sim \frac{\delta^2 S_{SUGRA}}{\delta g^{00}(x) \delta \Phi(y)} = 0$.

In summary, we first will determine the polarizabilities c_F and c_T from the mass shifts of the meson in two different backgrounds using eq. (2.21) and eq. (2.22). Subsequently we will compute the correlators in eq. (2.25) for $\mathcal{N} = 4$ theory at finite temperature. Finally we will put the results together using eq. (2.24) to deduce the rate of momentum broadening.

3. Determining matching coefficients with mass shifts

3.1 Backgrounds dual to finite temperature and field strength

3.1.1 Finite temperature background

The gravity background dual to $\mathcal{N} = 4$ SYM theory at finite temperature is given by the AdS-Schwarzschild black hole with Lorentzian signature (see e.g. [54]). This background

is needed below both for calculating the necessary two-point correlators $\langle T^{00}T^{00} \rangle$ and $\langle \mathcal{O}_{F^2} \mathcal{O}_{F^2} \rangle$, as well as for obtaining the meson polarizability, c_T .

We make use of the coordinates of [43] to write the AdS-Schwarzschild background with Lorentzian signature as

$$ds^2 = \frac{w^2}{R^2} \left(-\frac{f^2}{\tilde{f}} dt^2 + \tilde{f} d\mathbf{x}^2 \right) + \frac{R^2}{w^2} (d\varrho^2 + \varrho^2 d\Omega_3^2 + dw_5^2 + dw_6^2), \quad (3.1)$$

with the metric $d\Omega_3^2$ of the unit 3-sphere, and

$$\begin{aligned} f(r) &= 1 - \frac{r_H^4}{4w^4}, & \tilde{f}(r) &= 1 + \frac{r_H^4}{4w^4}, & w^2 &= \varrho^2 + w_5^2 + w_6^2, \\ r_H &= T\pi R^2, & R^4 &= 4\pi g_s N \ell_s^4, & \lambda &= 4\pi g_s N, & g_{YM}^2 &= g_s. \end{aligned} \quad (3.2)$$

This spacetime has a horizon at w_H which is determined by r_H as

$$w_H = \frac{r_H}{\sqrt{2}}, \quad (3.3)$$

and the boundary is reached at asymptotically large w . With $r_H = 0$ we obtain $AdS_5 \times S^5$. In section 4 we will work in a coordinate system with inverted radial AdS coordinate u used in e.g. [54]. In these coordinates, the metric reads

$$ds^2 = \frac{(\pi TR)^2}{u} (-f(u) dt^2 + d\mathbf{x}^2) + \frac{R^2}{4u^2 f(u)} du^2 + R^2 d\Omega_5^2, \quad (3.4)$$

with $f(u) = 1 - u^2$ and $d\Omega_5^2$ the unit 5-sphere metric.

3.1.2 Dilaton background

A non-trivial dilaton background that is dual to a field configuration with $\langle \mathcal{O}_{F^2} \rangle \neq 0$ and $\langle T^{\mu\nu} \rangle = 0$ has been given by Liu and Tseytlin [49] and consists of a configuration of D3 branes with homogeneously distributed D(-1) instantons. The type IIB action in the Einstein frame for the dilaton Φ , the axion C , and the self-dual gauge field strength $F_5 = \star F_5$ reads

$$S_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[\mathcal{R} - \frac{1}{2} (\partial\Phi)^2 - \frac{1}{2} e^{2\Phi} (\partial C)^2 - \frac{1}{4 \cdot 5!} (F_5)^2 + \dots \right]. \quad (3.5)$$

The ten-dimensional Newton constant is

$$\frac{1}{2\kappa_{10}^2} = \frac{1}{(2\pi)^7 \ell_s^8 g_s^2} = \frac{N^2}{4\pi^5 R^8}. \quad (3.6)$$

Solving the equations of motion derived from (3.5), Liu and Tseytlin [49] obtain the metric

$$ds_{\text{string}}^2 = e^{\Phi/2} ds_{\text{Einstein}}^2 = e^{\Phi/2} \left[\left(\frac{r}{R} \right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \left(\frac{R}{r} \right)^2 (dr^2 + r^2 d\Omega_5^2) \right]. \quad (3.7)$$

and axion-dilaton solution²

$$e^\Phi = 1 + \frac{q}{r^4}, \quad C = -i(e^{-\Phi} - 1). \quad (3.8)$$

The expectation value $\langle \mathcal{O}_{F^2} \rangle$ is given by

$$\langle \mathcal{O}_{F^2}(\vec{x}) \rangle = \lim_{r \rightarrow \infty} \frac{\delta S_{\text{IIB}}}{\delta \Phi(r, \vec{x})} = \frac{N^2}{2\pi^2 R^8} q. \quad (3.9)$$

3.2 Computing the polarization coefficients from meson mass shifts

Heavy mesons are identified with fluctuations $\tilde{\varphi}$ of a D7 brane embedded into the background dual to the field theory under consideration. Stable embeddings are obtained if the D7 brane spans all Minkowski directions as well as the radial AdS coordinate and a 3-sphere in the remaining polar directions. Consider the metric (3.1) as an example. The D7 brane spans all directions except w_5 and w_6 . The meson mass M is then obtained by solving the equation of motion for the fluctuations $\tilde{\varphi}$ [42]. Read as an eigenvalue equation, the equation of motion gives the meson mass as the eigenvalues M to the corresponding eigenfunctions $\tilde{\varphi}$. The discrete values of M describe the Kaluza-Klein mass spectrum of mesons for any given quark mass.

To see how this works, we outline this procedure for the vacuum case $\langle T^{00} \rangle = \langle \mathcal{O}_{F^2} \rangle = 0$, for which the meson spectrum was originally calculated in [42]. Subsequently we will introduce a non-zero $\langle \mathcal{O}_{F^2} \rangle$ and $\langle T^{00} \rangle$, respectively.

In the case of a D7 brane embedded in a ten dimensional background, the brane embedding is described by the location in the two directions transverse to the brane. We call these directions w_5 and w_6 . In general these locations depend on all eight coordinates ξ^i of the eight dimensional D7 brane worldvolume and are determined by extremizing the DBI-action

$$S_{\text{DBI}} = -T_7 \int d^8 \xi e^{-\Phi} \sqrt{-\det h}, \quad h_{ab} = \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b} G_{\mu\nu}^{\text{st}}, \quad (3.10)$$

where T_7 is the D7 brane tension and G^{st} is the string frame metric of the ten dimensional background with coordinates X^μ . It is related to the Einstein metric as in eq. (3.7). The distinction between the Einstein and string frame is ultimately important below. The pullback h contains the functions $w_5(\xi)$ and $w_6(\xi)$, which are determined by solving their equations of motion, derived from S_{DBI} .

The background $AdS_5 \times S^5$ dual to $\langle T^{00} \rangle = \langle \mathcal{O}_{F^2} \rangle = 0$ is obtained e.g. from (3.1) with $r_H = 0$. It is well known that for this background a probe brane embedding is given by the functions

$$w_5 = 0, \quad (3.11)$$

$$w_6 = L, \quad (3.12)$$

²For the conventions note that in our notation $q = \frac{R^8}{\lambda} q_{LT}$, where q_{LT} is used in the paper of Liu and Tseytlin [49].

where w_5 and w_6 are the coordinates given in (3.1) and the constant L determines the quark mass $m_q = L/(2\pi\ell_s^2)$. Now we allow for small fluctuations $\tilde{\varphi}_5$ and $\tilde{\varphi}_6$ around this solution,

$$w_5 = 2\pi\ell_s^2 \tilde{\varphi}_5(\vec{x}, \varrho) , \tag{3.13}$$

$$w_6 = L + 2\pi\ell_s^2 \tilde{\varphi}_6(\vec{x}, \varrho) . \tag{3.14}$$

By the symmetries of the setup, the fluctuations only depend on the Minkowski directions \vec{x} and on the coordinate ϱ , denoting the radial coordinate on the part of the D7 brane which is transverse to the Minkowski directions. The resulting equations of motion are identical for $\tilde{\varphi}_5$ and $\tilde{\varphi}_6$. Using φ to denote any one of them, it was shown in [42] that a solution to the equations of motion may be obtained by separating variables with the ansatz

$$\tilde{\varphi} = \varphi(\varrho) e^{i\vec{k}\vec{x}} \mathcal{Y}^\ell(S^3) , \tag{3.15}$$

where $\mathcal{Y}^\ell(S^3)$ are the scalar spherical harmonics on the S^3 wrapped by the probe D7 brane and \vec{k} denotes a four vector. The resulting equation of motion for the radial function $\varphi(\varrho)$ may be obtained from (3.10). For $\ell = 0$ it reads as

$$-\partial_\rho \rho^3 \partial_\rho \varphi(\rho) = \bar{M}^2 \frac{\rho^3}{(\rho^2 + 1)^2} \varphi(\rho) . \tag{3.16}$$

Here we introduced the following dimensionless quantities

$$\rho = \frac{\varrho}{L} , \quad \bar{M} = \frac{R^2}{L} M , \quad \frac{L}{R^2} = \frac{2\pi m_q}{\sqrt{\lambda}} , \tag{3.17}$$

and identified the meson mass squared M^2 with the square of the momentum four-vector \vec{k} of the fluctuations,

$$M^2 = -\vec{k}^2 . \tag{3.18}$$

The eigenfunctions φ_n solving the Sturm-Liouville equation (3.16) are given in terms of the standard hypergeometric function ${}_2F_1$,

$$\varphi_n(\rho) = \frac{c_n}{(\rho^2 + 1)^{n+1}} {}_2F_1(-n-1; -n; 2; -\rho^2) , \tag{3.19}$$

where c_n is a normalization constant such that

$$\int_0^\infty d\rho \frac{\rho^3}{(\rho^2 + 1)^2} \varphi_n(\rho) \varphi_m(\rho) = \delta_{nm} . \tag{3.20}$$

The lowest mode φ_0 is given by

$$\varphi_0(\rho) = \frac{\sqrt{12}}{\rho^2 + 1} . \tag{3.21}$$

The corresponding eigenvalues M_n to the functions φ_n are given by

$$\bar{M}_n = 2\sqrt{(n+1)(n+2)} . \tag{3.22}$$

We note that the mass of the lowest state with $n = 0$ is

$$M_0 = \frac{L}{R^2} 2\sqrt{2} = \frac{2\pi m_q}{\sqrt{\lambda}} 2\sqrt{2}, \quad (3.23)$$

which will appear frequently below. For a more detailed derivation of these results the reader is referred to [42].

3.2.1 Mass shift in the dilaton background

Let us now calculate the polarizability c_F which determines the change δM of the meson mass at a given value of the gauge condensate $\langle \mathcal{O}_{F^2} \rangle$ with respect to the meson mass at $\langle \mathcal{O}_{F^2} \rangle = 0$,

$$\delta M = -\frac{c_F}{N^2} \langle \mathcal{O}_{F^2} \rangle. \quad (3.24)$$

To find c_F we will determine the mass shift δM and identify c_F with the proportionality constant in front of $\langle \mathcal{O}_{F^2} \rangle$.

We are interested in the eigenvalues of fluctuations in the case of $q \propto \langle \mathcal{O}_{F^2} \rangle \neq 0$. The ten-dimensional background geometry dual to this scenario is given in (3.7) and the equation of motion for D7 brane fluctuations analog to (3.16) was derived in [55] to be

$$-\partial_\rho \rho^3 \partial_\rho \varphi(\rho) = \bar{M}^2 \frac{\rho^3}{(\rho^2 + 1)^2} \varphi(\rho) - 4\bar{q} \frac{\rho^4}{(\rho^2 + 1)(\bar{q} + (\rho^2 + 1)^2)} \partial_\rho \varphi(\rho), \quad (3.25)$$

with the dimensionless \bar{q}

$$\bar{q} = \frac{q}{L^4}. \quad (3.26)$$

To obtain analytical results, we consider the case of small \bar{q} and linearize in this parameter. Therefore the equation of motion to solve is

$$-\partial_\rho \rho^3 \partial_\rho \varphi(\rho) = \bar{M}^2 \frac{\rho^3}{(\rho^2 + 1)^2} \varphi(\rho) + \Delta(\rho) \varphi(\rho), \quad (3.27)$$

where the operator $\Delta(\rho)$ is given by

$$\Delta(\rho) = -4\bar{q} \frac{\rho^4}{(\rho^2 + 1)^3} \partial_\rho. \quad (3.28)$$

It is this term that describes the difference between the equation of motion at non vanishing background perturbation to (3.16), which is valid for $q = 0$.

To find the solution $\varphi_0(\rho)$ corresponding to the lightest meson with $n = 0$ we set up a perturbative expansion. Any deviation $\delta\varphi_0$ from the solution φ_0 of the case $q = 0$ may be written as a linear combination of the functions φ_n , which are a basis of the function space of all solutions,

$$\phi(\rho) = \phi_0(\rho) + \sum_{n=0}^{\infty} a_n \phi_n(\rho), \quad a_n \ll 1, \quad (3.29)$$

$$\bar{M}^2 = \bar{M}_0^2 + \delta\bar{M}_0^2, \quad \delta\bar{M}_0^2 \ll 1. \quad (3.30)$$

Plug this ansatz into the equation of motion (3.25), make use of (3.16) and keep terms up to linear order in the small parameters a_n , \bar{q} and δM_0^2 to get

$$\frac{\rho^3}{(\rho^2 + 1)^2} \sum_{n=0}^{\infty} a_n \bar{M}_n^2 \varphi_n(\rho) = \delta \bar{M}_0^2 \frac{\rho^3}{(\rho^2 + 1)^2} \varphi_0(\rho) + \bar{M}_0^2 \frac{\rho^3}{(\rho^2 + 1)^2} \sum_{n=0}^{\infty} a_n \varphi_n(\rho) + \Delta(\rho) \varphi_0(\rho). \quad (3.31)$$

We now multiply this equation by $\varphi_0(\rho)$, integrate over $\rho \in [0, \infty]$ and make use of (3.20) and (3.21) to see that

$$\begin{aligned} \delta \bar{M}_0^2 &= - \int_0^{\infty} d\rho \varphi_0(\rho) \Delta(\rho) \varphi_0(\rho) \\ &= -\frac{8}{5} \bar{q}. \end{aligned} \quad (3.32)$$

From $\delta \bar{M}_0^2 = 2\bar{M}_0 \delta \bar{M}_0$ we obtain

$$\delta M_0 = \frac{L}{2R^2} \frac{\delta \bar{M}_0^2}{\bar{M}_0} = -\frac{8}{5\pi} \left(\frac{2\pi}{M_0} \right)^3 \frac{1}{N^2} \langle \mathcal{O}_{F^2} \rangle, \quad (3.33)$$

where in the last step we used (3.23) for the mass and (3.9) and (3.26) to relate \bar{q} and \mathcal{O}_{F^2} . By comparison with (3.24) we identify the polarizability

$$c_F = \frac{8}{5\pi} \left(\frac{2\pi}{M_0} \right)^3. \quad (3.34)$$

3.2.2 Mass shift in the finite temperature background

The calculation of the polarizability c_T is completely analogous. We are now looking for the proportionality constant of meson mass shifts with respect to deviations from zero temperature,

$$\delta M = -\frac{c_T}{N^2} \langle T^{00} \rangle. \quad (3.35)$$

The background dual to the finite temperature field theory is the AdS black hole background given in (3.2). Notice that the black hole radius r_H is related to the expectation value $\langle T^{00} \rangle$ by [56]

$$\langle T^{00} \rangle = \frac{3}{8} \pi^2 N^2 T^4, \quad r_H = \pi T R^2. \quad (3.36)$$

Again we calculate the meson mass spectrum to identify the polarizability by comparison with (3.35). The embedding functions $w_5(\varrho)$ and $w_6(\varrho)$ in this background are given by

$$w_5 = 0, \quad (3.37)$$

$$w_6 = w_6(\varrho), \quad (3.38)$$

where the quark mass is determined by $m_q = \lim_{\varrho \rightarrow \infty} w_6 / (2\pi \ell_s^2)$. The function $w_6(\varrho)$ has to be computed numerically [43]. Some examples of such embeddings are shown in figure 2. We introduce small fluctuations $\varphi(\rho) e^{i\vec{k}\vec{x}}$ in the w_5 direction,

$$w_5 \rightarrow w_5(\varrho, \vec{x}) = \varphi(\varrho) e^{i\vec{k}\vec{x}}. \quad (3.39)$$

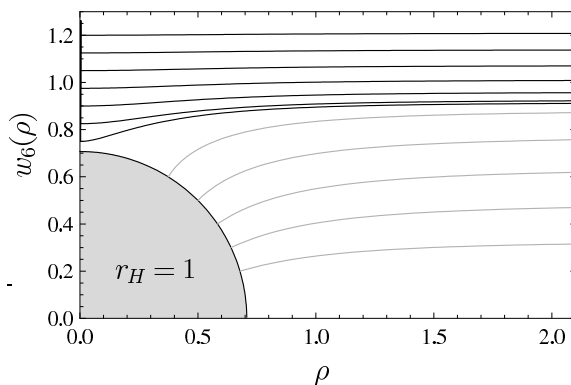


Figure 2: Some embedding functions $w_6(\rho)$ for different quark masses $m_q = \lim_{\rho \rightarrow \infty} w_6 / (2\pi\ell_s^2)$. For a detailed discussion see e.g. [43].

The linearized equation of motion for the fluctuations $\varphi(\varrho)$ in the limit of vanishing spatial momentum and $M^2 = -\vec{k}^2$ can be derived from the DBI action (3.10) to be

$$0 = \partial_\varrho \left[\mathcal{G} \sqrt{\frac{1}{1 + (\partial_\varrho w_6)^2}} \partial_\varrho \varphi(\varrho) \right] - \sqrt{1 + (\partial_\varrho w_6)^2} \frac{\varrho^3}{2(\varrho^2 + w_6^2)^5} r_H^8 \varphi(\varrho) + \mathcal{G} \sqrt{1 + (\partial_\varrho w_6)^2} \frac{4(\varrho^2 + w_6^2)^2 + r_H^4}{((\varrho^2 + w_6^2)^2 - r_H^4)^2} 4R^4 M^2 \varphi(\varrho), \quad (3.40)$$

where we abbreviated

$$\mathcal{G} = \varrho^3 \left(1 - \frac{r_H^8}{16(\varrho^2 + w_6^2)^4} \right). \quad (3.41)$$

In the regime of small temperatures, we may linearize in r_H^4 which is the leading order in r_H . Furthermore, as may be seen from figure 2, in the regime of a small temperature T compared to the quark mass m_q , or respectively small ratios of $r_H / \lim_{\rho \rightarrow \infty} w_6(\rho)$, the embeddings become more and more constant. So for constant embeddings $w_6 = L$ and up to order $T^4 \propto r_H^4$ the equation of motion simplifies to

$$-\partial_\rho \rho^3 \partial_\rho \varphi(\rho) = \bar{M}^2 \frac{\rho^3}{(\rho + 1)^2} \varphi(\rho) + \Delta(\rho) \varphi(\rho), \quad (3.42)$$

where we made use of the dimensionless quantities (3.26) and identify

$$\Delta(\rho) = \frac{3 r_H^4}{4 L^4} \frac{\rho^3}{(\rho^2 + 1)^4} \bar{M}^2. \quad (3.43)$$

For the lightest meson, the ansatz (3.29) this time leads to

$$\begin{aligned} \delta \bar{M}_0^2 &= - \int_0^\infty d\rho \varphi_0(\rho) \Delta(\rho) \varphi_0(\rho) \\ &= - \frac{9}{40} \frac{r_H^4 \bar{M}_0^2}{L^4}. \end{aligned} \quad (3.44)$$

Reinstating units and solving for δM_0 leads to

$$\delta M_0 = -\frac{12}{5\pi} \left(\frac{2\pi}{M_0} \right)^3 \frac{1}{N^2} \langle T^{00} \rangle . \quad (3.45)$$

From this we can read off the polarizability c_T as

$$c_T = \frac{12}{5\pi} \left(\frac{2\pi}{M_0} \right)^3 . \quad (3.46)$$

4. Finite Temperature Correlators

According to eq. (2.24) we need to compute the following correlators at finite temperature

$$G_R^{TT}(\omega, \mathbf{q}) = -i \int d^4x e^{+i\omega t - i\mathbf{q}\cdot\mathbf{x}} \theta(t) \langle [T^{00}(\mathbf{x}, t), T^{00}(\mathbf{0}, 0)] \rangle , \quad (4.1)$$

$$G_R^{FF}(\omega, \mathbf{q}) = -i \int d^4x e^{+i\omega t - i\mathbf{q}\cdot\mathbf{x}} \theta(t) \langle [\mathcal{O}_{F^2}(\mathbf{x}, t), \mathcal{O}_{F^2}(\mathbf{0}, 0)] \rangle . \quad (4.2)$$

The calculational procedure for these two correlators is standard and has been discussed in [58, 57]. T^{00} correlators are associated with graviton propagation and \mathcal{O}_{F^2} correlators are associated with dilaton propagation.

On the gravity side both field correlators are computed in the black hole background (3.1) placing the dual gauge theory operator correlation functions at finite temperature. For simplicity, in this section we work in the conventions and coordinates of [54]. We apply the method developed in [58, 57] and first applied in [54], in order to find the two-point Minkowski correlators as

$$G^R(\omega, \mathbf{q}) = A(u) f(u, -\vec{k}) \partial_u f(u, \vec{k}) \Big|_{u \rightarrow 0} . \quad (4.3)$$

The function $f(u, \vec{k})$ relates the boundary and bulk values of a gravity field to each other. For example the dilaton field Φ is related to its value at the boundary ϕ^{bdy} by

$$\Phi(u, \vec{k}) = f(u, \vec{k}) \phi^{\text{bdy}}(\vec{k}) , \quad (4.4)$$

and is normalized to one at the boundary $f(0, \vec{k}) = 1$. For metric fluctuations, $\Phi(u, \vec{k})$ is replaced by $h^{00}(u, \vec{k})$. The factor $A(u)$ can be read off from the classical supergravity action

$$S_{\text{cl}} = \frac{1}{2} \int du d^4x A(u) (\partial_u \Phi)^2 + \dots . \quad (4.5)$$

The classical gravity action for the graviton and dilaton is obtained from (3.5) as

$$S = \frac{1}{2\kappa_5^2} \int du d^4x \sqrt{-g_5} \left[(\mathcal{R} - 2\Lambda) - \frac{1}{2} (\partial\Phi)^2 + \dots \right] , \quad (4.6)$$

where

$$\frac{1}{\kappa_5^2} = \frac{R^5 \Omega_5}{\kappa_{10}^2} = \frac{N^2}{4\pi^2 R^3} . \quad (4.7)$$

So comparing to (4.5) we get

$$A_{\Phi} = -\frac{1}{2\kappa_5^2} \sqrt{-g_5} g^{uu} . \quad (4.8)$$

The equation of motion derived from (4.6) in momentum space reads

$$\Phi'' - \frac{1+u^2}{uf(u)} \Phi' + \frac{\mathfrak{w}^2 - \mathfrak{q}^2 f(u)}{uf(u)^2} \Phi = 0 , \quad (4.9)$$

with the function $f(u) = 1 - u^2$, the dimensionless frequency $\mathfrak{w} = \omega/2\pi T$ and spatial momentum component $\mathfrak{q} = q/2\pi T$. The equation of motion (4.9) has to be solved numerically with incoming wave boundary condition at the black hole horizon. Computing the indices and expansion coefficients near the boundary as done in [59, 60], we obtain the asymptotic behavior as linear combination of two solutions

$$\Phi(u) = (1 + \dots) + \mathcal{B}(u^2 + \dots) , \quad (4.10)$$

where \mathcal{B} is the coefficient for the second solution and the coefficient for the first solution has been set to 1. At the horizon the asymptotic solution satisfying the incoming wave boundary condition is

$$\Phi(u) = (1 - u)^{-i\mathfrak{w}/2} (1 + \dots) . \quad (4.11)$$

As discussed in [59, 60] we find the coefficient \mathcal{B} by integrating the two boundary solutions from (4.10) forward towards the horizon and by matching the linear combination of the numerical solutions $\Phi(u) = \Phi_1^{\text{num}} + \mathcal{B}\Phi_2^{\text{num}}$ to the solution (4.11) at the horizon. The imaginary part of the retarded correlator then is given by

$$\frac{-2T}{\omega} \text{Im} G_{\Phi\Phi}^R = \frac{N^2(\pi T)^4}{4\pi^2} \frac{2}{\pi} \frac{\text{Im}\mathcal{B}}{\mathfrak{w}} . \quad (4.12)$$

Solving (4.9) and matching the asymptotic solutions as described above, we obtain

$$\lim_{\omega \rightarrow 0} \int \frac{d^3\mathfrak{q}}{(2\pi)^3} \frac{\mathfrak{q}^2}{3} \left[\frac{-2T}{\omega} \text{Im} G_{F^2 F^2}^R(\omega, \mathfrak{q}) \right] = N^2 T^9 67.258 . \quad (4.13)$$

The corresponding result for the energy-momentum tensor correlator is obtained in an analogous way but the analysis is significantly more complicated. Fortunately it has been extensively and carefully analyzed [57]. The final result is

$$\lim_{\omega \rightarrow 0} \int \frac{d^3\mathfrak{q}}{(2\pi)^3} \frac{\mathfrak{q}^2}{3} \left[\frac{-2T}{\omega} \text{Im} G_{TT}^R(\omega, \mathfrak{q}) \right] = N^2 T^9 355.169 . \quad (4.14)$$

5. Summary and discussion

Over the duration of the lifetime of the heavy meson state the meson will loose momentum on average and simultaneously receive random kicks as codified by the Langevin equations of motion

$$\frac{dp^i}{dt} = -\eta_D p^i + \xi^i(t) , \quad \langle \xi^i(t) \xi^j(t') \rangle = \kappa \delta^{ij} \delta(t - t') . \quad (5.1)$$

The drag and momentum broadening rates are related by the Einstein relation

$$\eta_D = \frac{\kappa}{2TM_0}, \quad (5.2)$$

with M_0 the meson mass. Collecting the results for polarizabilities (3.34), (3.46) and force correlators (4.13), (4.14), and using eq. (2.24) we obtain our principal result

$$\begin{aligned} \kappa &= \frac{T^3}{N^2} \left(\frac{2\pi T}{M_0} \right)^6 \left[\left(\frac{8}{5\pi} \right)^2 (67.258) + \left(\frac{12}{5\pi} \right)^2 (355.169) \right] \\ &= \frac{T^3}{N^2} \left(\frac{2\pi T}{M_0} \right)^6 [224.726] . \end{aligned} \quad (5.3)$$

The finite temperature mass shift in $\mathcal{N} = 4$ SYM is

$$\begin{aligned} \delta M_0 &= -\frac{cT}{N^2} \langle T^{00} \rangle \\ &= -T \left(\frac{2\pi T}{M_0} \right)^3 \frac{9\pi}{10} . \end{aligned} \quad (5.4)$$

Comparing these formulas with the analogous formulas in weak coupling large N QCD given in Eqs. (2.18) and (2.5),

$$\kappa_{\text{pQCD}} = \frac{T^3}{N^2} \left(\frac{\pi T}{\Lambda_B} \right)^6 \frac{50176}{1215} \pi, \quad (5.5)$$

and

$$\delta M_{\text{pQCD}} = -T \left(\frac{\pi T}{\Lambda_B} \right)^3 \frac{14}{45}, \quad (5.6)$$

we see that the meson mass M_0 plays the role of the inverse Bohr radius $\Lambda_B = (m_q/2)\alpha_s C_F$ in the strong coupling dipole effective Lagrangian. This is as expected for relativistic bound states. Below we will compare the values of the ratio $\kappa/(\delta M^2)$ at strong and weak coupling.

Perhaps the theoretically most important aspect of this work is that we have deduced a drag coefficient which is suppressed by N^2 in the large N limit. On the field theory side, this was achieved by calculating the mass shift of a meson in an external background (which is finite at large N), and using this information to deduce the meson couplings to the stress tensor and the operator \mathcal{O}_{F^2} . We have restricted the calculation to the heavy dipole limit where these are the only relevant operators. Subsequently, the fluctuations of these operators give rise to a net force on the meson.

On the gravity side, the meson mass shift arises as a change in the normal vibrational modes of the D7 brane in the presence of an external gravitational (or dilatonic) field. Although it is not manifest in the usual black hole finite temperature AdS/CFT setup, the gravitational field and dilatonic fields are continually fluctuating. This is encoded by the fluctuation dissipation theorem in the field theory which does emerge in a Kruskal formalism of the gauge gravity duality (The fluctuation dissipation theorem was used to relate eq. (2.10) and eq. (2.12)). Since these fluctuating gravitational and dilatonic fields shift the spectrum of the D7 brane excitations, gradients in these fields give rise to a net force on a

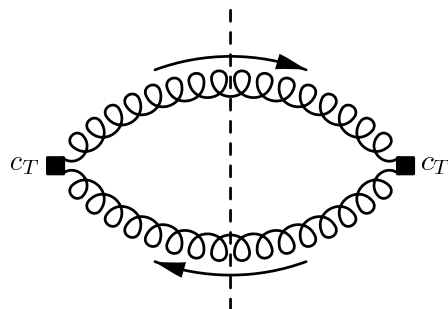


Figure 3: Feynman graph leading to scattering of a heavy meson in perturbation theory. This graph represents the gauge contribution to correlators given in eq. (5.7). At strong coupling the effect of additional scatterings is to reduce the integrated value of this correlator by almost a factor of five in the appropriate kinematic regime.

mesonic normal modes of the D7 brane. This discussion suggests a better understanding of how gravitational and dilatonic fields fluctuate in bulk, would give a straightforward procedure to calculate the drag of a finite mass meson. Specifically, fluctuations in the bulk would force motion of meson wave functions which extend into the fifth dimension. We hope to pursue this reasoning in the future.

From a phenomenological perspective the current calculation was limited to very heavy mesons (which survive above T_c) where dipole interactions between the meson and the medium are dominant. It is certainly unclear if this is the relevant interaction mechanism above T_c even for bottomonium. Furthermore, the dipole coupling between a heavy meson and the medium is dominated by short distance physics which is not well modeled by AdS/CFT.

However, after the gluons scatter off the heavy quark, they propagate out into the plasma which modifies the free propagation as indicated by the Feynman graph in figure 3. To factorize this long distance dynamics from the short distance meson dynamics we form the ratio

$$\begin{aligned} \left[\frac{\kappa}{(\delta M)^2} \right]_{\lambda \rightarrow \infty} &\simeq \frac{-1}{\langle (T^{00}) \rangle^2} \int_{-\infty}^{\infty} dt \nabla_{\mathbf{y}}^2 \langle T^{00}(\mathbf{y}, t) T^{00}(\mathbf{x}, 0) \rangle \Big|_{\mathbf{y}=\mathbf{x}}, \\ &\simeq \frac{\pi T}{N^2} 8.25, \end{aligned} \tag{5.7}$$

which is independent of the short distance coefficients c_T and c_F provided the numerically small dilatonic contribution is neglected³. It is then reasonable to use AdS/CFT to estimate to what degree strong coupling physics modifies this ratio in QCD. In the free finite temperature $\mathcal{N} = 4$ theory, the result is (see appendix A)

$$\left[\frac{\kappa}{(\delta M)^2} \right]_{\lambda \rightarrow 0} \simeq \frac{\pi T}{N^2} 37.0, \tag{5.8}$$

Thus comparing the strong coupling result (5.7) with the weak-coupling result (5.8), we conclude that strong coupling effects actually *reduce* the scattering rate relative to the

³With the dilaton contribution the coefficient is 8.95.

mass shift. Roughly speaking, the same strong coupling physics that is responsible for the reduction of pressure (by a factor of 3/4) relative to the Stefan-Boltzmann prediction is at work here. However the effect is more pronounced since the correlator in eq. (5.7) is dominated by larger values of spatial momentum \mathbf{q} .

Given this AdS/CFT result we expect the perturbative estimates for the rate of momentum broadening to be *reduced* by some factor which could be as large as a factor of five. We will not speculate on this factor here but simply write the perturbative momentum diffusion rate as

$$\kappa_{\text{QCD}} = T(\delta M)^2 \frac{1280\pi}{3N^2}, \quad (5.9)$$

with a gravitationally biased opinion that the coefficient is too large. To obtain a numerical estimate for the $\Upsilon(1s)$ state, we take $T = 340 \text{ MeV} \simeq 2T_c$, $M_0 = 9.46 \text{ GeV}$, and estimate the mass shift as $\delta M \simeq -10 \text{ MeV}$ based on potential model calculations which fit lattice data [39, 40]. Substituting into eq. (5.9) and eq. (5.2) for the relaxation time $\tau_R \equiv \eta_D^{-1}$ we find

$$\kappa_{\text{QCD}} = 0.025 \frac{\text{GeV}^2}{\text{fm}} \left(\frac{T}{340 \text{ MeV}} \right) \left(\frac{\delta M_0}{10 \text{ MeV}} \right)^2, \quad (5.10)$$

$$\tau_R \equiv \frac{1}{\eta_D} = 250 \text{ fm} \left(\frac{M_0}{9.46 \text{ GeV}} \right) \left(\frac{10 \text{ MeV}}{\delta M_0} \right)^2. \quad (5.11)$$

The introduction discusses some of the significant uncertainties associated with the current formalism and the extraction of mass shift adopted here. Nevertheless, the AdS/CFT prediction is that strong coupling effects will actually increase this perturbative estimate of the relaxation time by up to a factor of five.

Acknowledgments

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A. Diffusion rate in perturbation theory

The purpose of this appendix is to compute $\kappa/(\delta M)^2$ in free finite temperature field theory in $\mathcal{N} = 4$ Super Yang Mills theory. This will permit a comparison to the strongly interacting results.

A.1 QCD computation

In the interest of pedagogy we will indicate in detail how the QCD computation is performed. We will work in the limit where only the coupling to the electric field is included

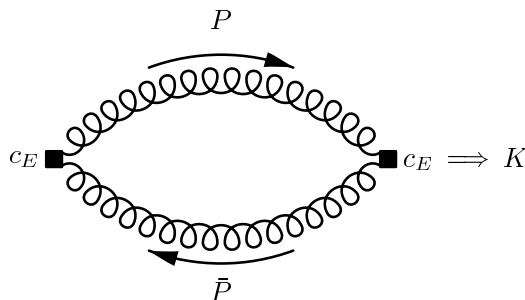


Figure 4: Euclidean graph corresponding to the eq. (2.14) and eq. (A.3).

i.e., $c_B = 0$. κ is given by eq. (2.14). We will use the Matsubara formalism though the real time formalism is not more difficult in this case. We will work in the Coulomb gauge where the propagators are

$$\int_0^\beta d\tau \int d^3\mathbf{x} e^{-iK \cdot X} \langle A_0(X) A_0(0) \rangle = \frac{1}{k^2}, \tag{A.1}$$

$$\int_0^\beta d\tau \int d^3\mathbf{x} e^{-iK \cdot X} \langle A_i(X) A_j(0) \rangle = \frac{\hat{k}_i \hat{k}_j - \delta_{ij}}{K^2}. \tag{A.2}$$

Here we follow standard thermal field theory notation $K^\mu = (\omega_n, \mathbf{k})$ and $X^\mu = (\tau, \mathbf{x})$ with $k = |\mathbf{k}|$. $\omega_n = 2\pi nT$ labels the Matsubara and $K \cdot X = \omega_n \tau + \mathbf{k} \cdot \mathbf{x}$. The Euclidean metric is $g_E^{\mu\nu} = \text{diag}(+, +, +, +)$ and further explanation of Euclidean conventions is given in ref. [61]. Then the Euclidean correlator corresponding to eq. (2.14) and figure 4 for a single color index is

$$G_E^{\mathcal{E}^2 \mathcal{E}^2}(K) = \frac{1}{2} T \sum_{P^0} \int \frac{d^3\mathbf{p}}{(2\pi)^3} P^0 \bar{P}^0 \frac{\hat{p}_i \hat{p}_j - \delta_{ij}}{P^2} P^0 \bar{P}^0 \frac{\hat{p}_j \hat{p}_i - \delta_{ji}}{\bar{P}^2} + \text{Coulomb graphs}. \tag{A.3}$$

Here $\bar{P} = P - K$ and we do not write the graphs involving coulomb lines since these do not contribute to the imaginary part. Performing the Matsubara sum, analytically continuing $-iK^0 \rightarrow \omega + i\epsilon$, taking the imaginary part, and finally working in the limit that $\omega \rightarrow 0$ yields the following result for the imaginary part of the retarded correlator,

$$\begin{aligned} \lim_{\omega \rightarrow 0} \frac{-2T}{\omega} \text{Im} G_R^{\mathcal{E}^2 \mathcal{E}^2}(\omega, \mathbf{k}) & \tag{A.4} \\ &= \int \frac{d^3p}{(2\pi)^3 2E_{\mathbf{p}}} \frac{d^3\bar{p}}{(2\pi)^3 2E_{\bar{\mathbf{p}}}} n_{\mathbf{p}} (1 + n_{\mathbf{p}}) 2\pi \delta(E_{\mathbf{p}} - E_{\bar{\mathbf{p}}}) (2\pi)^3 \delta^3(\mathbf{p} - \bar{\mathbf{p}} - \mathbf{k}) |\mathcal{M}|^2, \end{aligned}$$

with

$$|\mathcal{M}|^2 = (E_{\mathbf{p}} E_{\bar{\mathbf{p}}})^2 (1 + \cos^2(\theta_{\mathbf{p}\bar{\mathbf{p}}})) . \tag{A.5}$$

The details of the preceding steps can be streamlined and are found in many places; see ref. [61] and the text book [62] for simple explanations. Integrating over the retarded correlator as required by eq. (2.14) and multiplying by N^2 to account for the number of

gluons yields the following result

$$\kappa = \frac{c_E^2}{N^2} \frac{64\pi^5}{135} T^9. \quad (\text{A.6})$$

This result is the expected kinetic theory result for the rate of momentum diffusion of a heavy meson scattering via dipole scattering.

A.2 $\mathcal{N} = 4$ computation

The free $\mathcal{N} = 4$ Lagrangian is written as follows:

$$\mathcal{L} = 2 \text{tr} \left\{ -\frac{1}{4} F^2 + \frac{1}{2} \bar{\lambda}_a (-i\bar{\sigma} \cdot \partial) \lambda^a - \frac{1}{2} \partial_\mu X_i \partial^\mu X_i \right\}, \quad (\text{A.7})$$

where “ a ” is a SU(4) index and “ i ” is a SO(6) index. Under flavor rotation, λ^a transforms in the fundamental representation of SU(4) and X^i transforms as the fundamental representation of SO(6). SU(4) and SO(6) are locally isomorphic. SU(4) matrices are parameterized as $e^{i\beta_A (T^4)_A}$ with trace normalization $\text{tr}[(T^4)_A (T^4)_B] = C_4 \delta_{AB}$ and $C_4 = 1/2$. Similarly, SO(6) matrices are written as $e^{i\beta_A (T^6)_A}$, with trace normalization $C_6 = 1$ [63]. The normalization convention adopted here has been fixed so that the AdS/CFT correspondence holds at the level of non-renormalized two point functions at zero temperature [64].

The full stress tensor is written

$$T^{\mu\nu} = (T^{\mu\nu})_{\text{gauge}} + (T^{\mu\nu})_{\text{fermion}} + (T^{\mu\nu})_{\text{scalar}}, \quad (\text{A.8})$$

with

$$(T^{\mu\nu})_{\text{gauge}} = 2 \text{tr} \left\{ F_\kappa^\mu F^{\nu\kappa} + g^{\mu\nu} \left(-\frac{1}{4} F^2 \right) \right\}, \quad (\text{A.9})$$

$$(T^{\mu\nu})_{\text{fermion}} = 2 \text{tr} \left\{ \frac{i}{8} \bar{\lambda} \left(\bar{\sigma}^\mu \overleftrightarrow{\partial}^\nu + \bar{\sigma}^\nu \overleftrightarrow{\partial}^\mu \right) \lambda + g^{\mu\nu} \left(\frac{1}{2} \bar{\lambda}^a (-i\bar{\sigma} \cdot \partial) \lambda_a \right) \right\}, \quad (\text{A.10})$$

$$(T^{\mu\nu})_{\text{scalar}} = 2 \text{tr} \left\{ \partial^\mu X_i \partial^\nu X_i + g^{\mu\nu} \left(-\frac{1}{2} \partial_\alpha X_i \partial^\alpha X_i \right) \right\}. \quad (\text{A.11})$$

Here $\overleftrightarrow{\partial} \equiv \overrightarrow{\partial} - \overleftarrow{\partial}$.

There are three graphs for the gauge, fermion, and scalar loops which make a contribution to the imaginary part of the retarded correlator. The full diffusion rate is

$$\kappa = (\kappa)_A + (\kappa)_\lambda + (\kappa)_X, \quad (\text{A.12})$$

where $(\kappa)_A$ is due to gauge bosons, $(\kappa)_\lambda$ is due to fermions and $(\kappa)_X$ is due to scalars. In each case the retarded correlator can be written in the form of a phase space integral times a matrix element squared. The matrix elements are

$$|\mathcal{M}|_A^2 = [N^2] E_{\mathbf{p}}^4 (1 + \cos(\theta_{\mathbf{p}\bar{\mathbf{p}}}))^2 (1 + \cos^2(\theta_{\mathbf{p}\bar{\mathbf{p}}})), \quad (\text{A.13})$$

$$|\mathcal{M}|_\lambda^2 = [4N^2] 4E_p^4 (1 + \cos(\theta_{\mathbf{p}\bar{\mathbf{p}}})), \quad (\text{A.14})$$

$$|\mathcal{M}|_X^2 = [6N^2] E_p^4 (1 + \cos \theta_{\mathbf{p}\bar{\mathbf{p}}})^2. \quad (\text{A.15})$$

Then integrating over the phase-space we obtain the three contributions to κ

$$\left(\frac{N^4}{c_T^2}\kappa\right)_A = [N^2]\frac{64\pi^5}{225}T^9, \quad (\text{A.16})$$

$$\left(\frac{N^4}{c_T^2}\kappa\right)_\lambda = [4N^2]\frac{254\pi^5}{135}T^9, \quad (\text{A.17})$$

$$\left(\frac{N^4}{c_T^2}\kappa\right)_X = [6N^2]\frac{32\pi^5}{135}T^9. \quad (\text{A.18})$$

The final result for the momentum diffusion rate when only the stress tensor coupling is included is

$$\left(\frac{N^4}{c_T^2}\kappa\right) = N^2\frac{6232\pi^5}{675}T^9. \quad (\text{A.19})$$

Similarly the mass shift for the meson in the finite temperature background is

$$\delta M = (\delta M)_A + (\delta M)_\lambda + (\delta M)_X. \quad (\text{A.20})$$

The different components of the mass shift are

$$(\delta M)_A = \frac{c_T}{N^2}[2N^2]\frac{\pi^2T^4}{30}, \quad (\text{A.21})$$

$$(\delta M)_\lambda = \frac{c_T}{N^2}\left[8N^2\frac{7}{8}\right]\frac{\pi^2T^4}{30}, \quad (\text{A.22})$$

$$(\delta M)_X = \frac{c_T}{N^2}[6N^2]\frac{\pi^2T^4}{30}. \quad (\text{A.23})$$

In each case the $\pi^2T^4/30$ is the energy density of a massless single component Bose gas. The factor in square brackets counts the number of degrees of freedom (including spin) and a factor of $7/8$ to account for the differences between Bose and Fermi distributions. Putting these pieces together we find the total mass shift due to coupling to the tensor

$$\delta M = c_T\frac{\pi^2T^4}{2}. \quad (\text{A.24})$$

Now we finally evaluate the ratio in the free theory

$$\begin{aligned} \frac{\kappa}{(\delta M)^2} &= \frac{24928}{675}\frac{\pi T}{N^2} \\ &\simeq 37.0\frac{\pi T}{N^2}. \end{aligned} \quad (\text{A.25})$$

This is the weak coupling expectation for this ratio provided the dominant coupling of the medium to the dipole is through the stress tensor operator. It is useful to compare this expectation to the strong coupling results as is done in the body of the text.

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